Supersonic Nozzle Design & Performance Analysis

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Application:

Commercial Sea Level Rocket

QoI #1:

Thrust Coefficient

QoI #2:

Specific Impulse

Synthesis

Aspects of Engineering	Pros	Cons
Experiments (Supersonic tunnel)	Captures real physics, shows transient behavior / expansion (under, over, or perfect)	Data is mostly qualitative, limited internal flow data, sensitive to setup conditions
Theory (Quasi-1D)	Fastest tool, replicable, steady state quick predictions of p, T, ρ, M, etc,	Carries several assumptions (steady state, 1D, idealized), can not capture 2D shocks or transient behavior
Computational Fluid Dynamics (MFC)	Validate flow structure beyond 1D metrics, applicable to transient observation (validating nozzle robustness)	Time consuming, computationally intensive, model assumptions can produce unrealistic results

Key Takeaways from FPC P1 / 2:

- Quasi-1D steady theory is best for optimization of geometry because nozzles can be directly compared at the same conditions without time-dependent (transient) conditions muddying results.
- Experiments and MFC are best for observing flow physics → Transient tools show what is happening physically, and speak to the robustness of our design (how it operates in startup, shutdown, states outside intended conditions), while steady-state models tell us what geometry is best under specific conditions (determined by our application).

Quantities of Interest

Thrust Coefficient

$$C_F = \frac{F}{p_c \, A_t}$$

Measures nozzle efficiency independent of size (scales up and down)

Specific Impulse

$$I_{sp} = \frac{C_F \cdot c^*}{g}$$

Measures how much thrust is produced per unit of propellant consumed

QoIs Considered [1,2,3]

1. Thrust
2. Thrust Coefficient
3. Mass flow
4. Specific Impulse
5. Exit Pressure
6. Ambient Pressure
7. Nozzle Efficiency

Takeaways

1. Both are maximized simultaneously due to their linear relationship
2. C_F is sensitive to wall angle and area ratio, thus optimizing relationship is important
3. Well-optimized design point gives you the best average performance across the full ascent, not just at one moment

[1] Tuttle, Perfect Bell Nozzle Parametric and Optimization Curves (NASA, 1983)
[2] Jones et al., Proposed Flight Research of a Dual-Bell Rocket Nozzle System to Enhance NASA's Space Launch System Performance (NASA, 2013)
[3] Kacynski et al., The Prediction of Nozzle Performance and Heat Transfer in Rocket Engines (NASA, 1994)

Nozzle Geometries

Geometry #1: Bell [1]

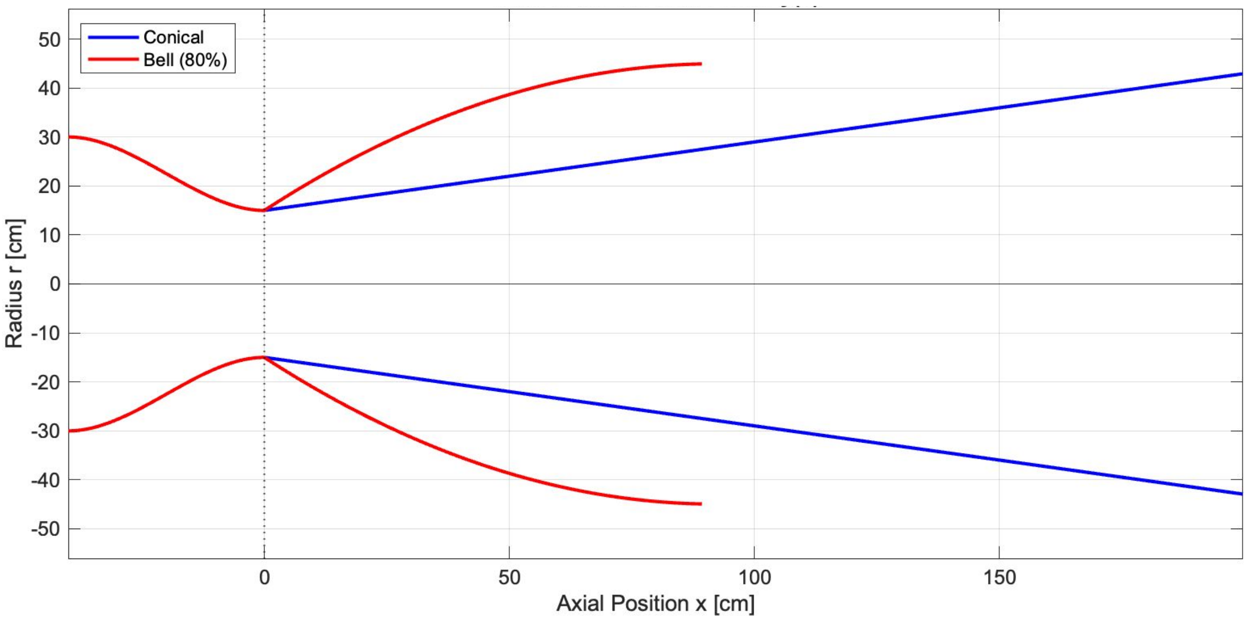
$$y(x) = ax^2 + \tan(\theta_N)\,x + r_t$$

Diverging section with parabolic wall profile where wall angle decreases continuously

Geometry #2: Conical [2]

$$y(x) = r_t + \tan(\alpha)\,x$$

Diverging section with straight wall profile

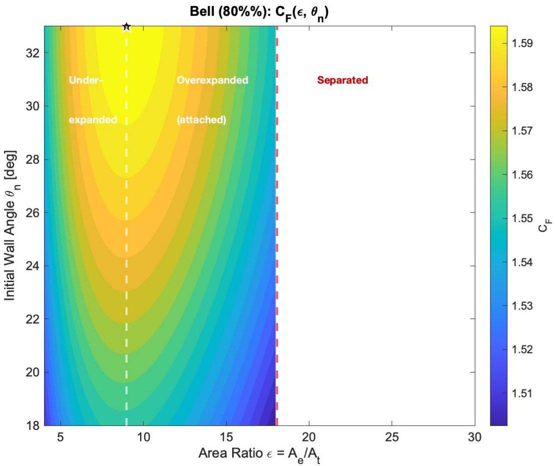


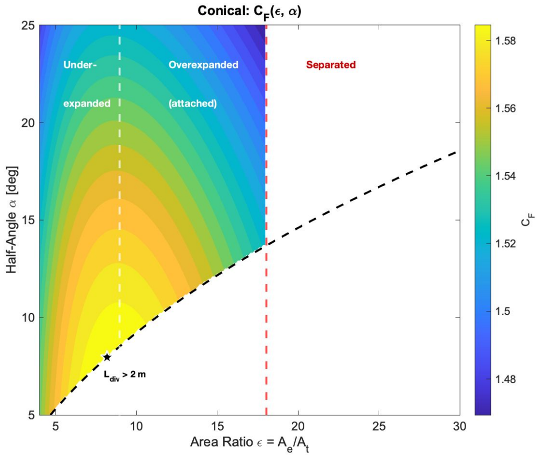
[1] “Rocket Nozzles: Types, Manufacturing & Materials,” Astroforge Tech. Available: https://astforgetech.com/rocket-nozzles-types-manufacturing-materials/
[2] G. V. R. Rao, “Exhaust Nozzle Contour for Optimum Thrust,” Jet Propulsion, 1958. Available: https://www.semanticscholar.org/paper/Exhaust-Nozzle-Contour-for-Optimum-Thrust-Rao/230e0a17e1a07550212b65f99f7e46aac59811ae
[3] Dieter K. Huzel and David H. Huang, Modern Engineering for Design of Liquid-Propellant Rocket Engines. American Institute of Aeronautics and Astronautics

Parameter Space and Performance

Optimization Results

	Parameter	Conical	Bell (80%)
Optimization	Area ratio $\varepsilon = A_e/A_t$	8.18	8.96
	Shape angle	$\alpha = 8.0°$	$\theta_N = 33.0°,\ \theta_E = 1.2°$
QoIs	Thrust coefficient C_F	1.590	1.598
	Specific impulse I_{sp} [s]	287.5	289.0
Engineering	Diverging length L_{div} [m]	1.997	0.893
	Thrust F [kN]	786.9	790.9



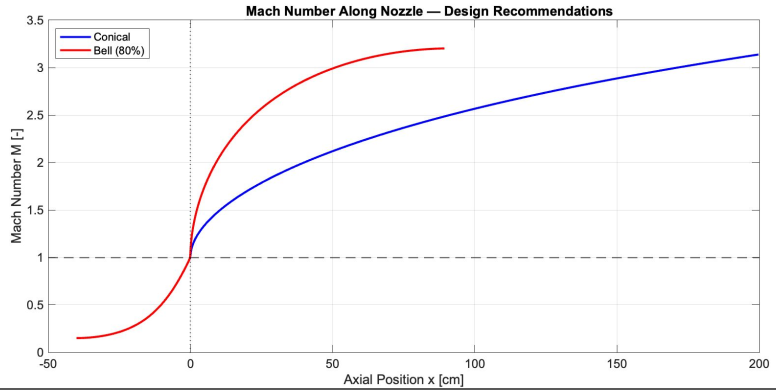


Initial Conditions[1]

Parameter	Symbol	Value	Units
Gas properties			
Ratio of specific heats	γ	1.25	—
Specific gas constant	R_g	355.4	J/kg·K
Chamber conditions			
Stagnation pressure	p_c	6.89	MPa
Stagnation temperature	T_c	3533	K
Ambient conditions			
Atmospheric pressure	p_{atm}	101.325	kPa
Fixed geometry			
Throat radius	r_t	0.15	m
Max diverging length	L_{max}	2.00	m

[1] "SpaceX Unveils Plans To Be World's Top Rocket Maker. Available: https://web.archive.org/web/20150621134912/http://aviationweek.com/awin/spacex-unveils-plans-be-world-s-top-rocket-maker

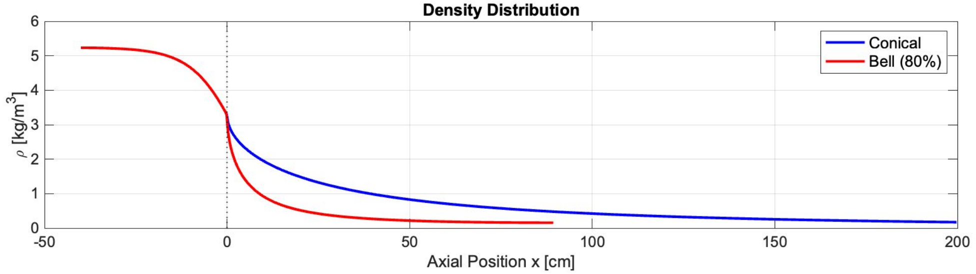
Ideal Performance

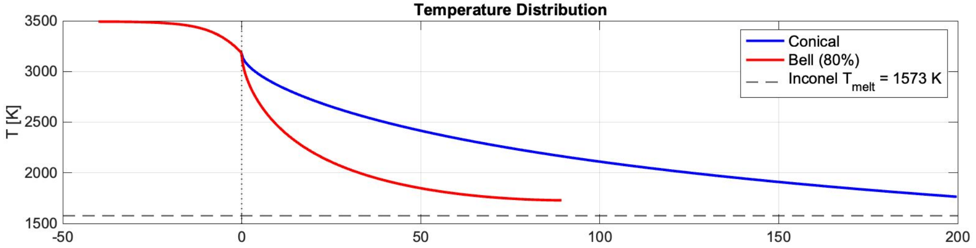


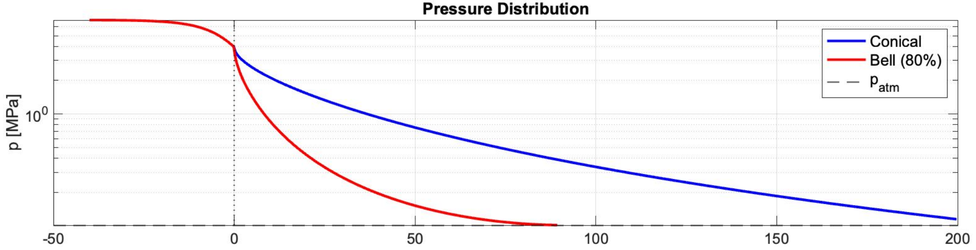
Flow properties	Exit Mach number M_e	3.138	3.202
	Exit pressure ratio p_e/p_{atm}	1.131	1.001
	Throat temperature T^* [K]	3182 (both)	
	Exit temperature T_e [K]	1764	1728

Bell beats conical by **+0.51%** in C_F and **+1.5s** in I_{sp} at **55%** shorter length.

Throat temperature reaches **3182K**, but Inconel 718 melts at **1573K**[1], thus wall would vaporize. We propose using regenerative cooling, which pumps cold fuel through wall channels before combustion to lower wall temperature.







[1] M. Abdullah K., Kavyassree G., and Srimathi S., “Comparative Analysis of Inconel 718 and Inconel 625 for Rocket Nozzles: Performance, Durability, and Manufacturing Influence,” Acceleron Aerospace Journal, vol. 4, no. 3, 2025. Available: https://acceleron.org.in/index.php/aaj/article/view/227

Recommendations

Overall Recommendation:

Bell Shaped Nozzle with the following form:

$$y(x) = -0.3521\,x^2 + \tan(33°)\,x + 0.15$$

The 80% bell shaped nozzle achieved higher C_F and I_{sp} at less than half the diverging length of the equivalent conical design.

Final Geometry

Area ratio $\varepsilon = A_e/A_t$	8.96
Initial wall angle θ_N	33.0°
Exit wall angle θ_E	1.2°
Diverging length L_{div}	0.893 m
Exit radius r_e	0.449 m

Recommendations and Reflections

1. The bell outperformed conical by only +0.51% in C_F, so maybe area ratio dominates performance at this pressure and wall shape is a secondary factor
2. Model how thrust changes at various altitudes, since our optimizer only finds the best design at one fixed altitude
3. Run 2D fluid simulations on the optimal bell nozzle to check how much real-world losses like friction and shocks reduce performance below our theoretical predictions

Contributions

Peyton Marcotte: Writing, Original Draft (lead), Visualization (equal).

Logan Suryamega: Formal Analysis (lead), Writing, Review, and Editing (equal).

Juan Perez: Data Collection (lead), Software / Code (equal).

All authors reviewed and approved the final submission.

MATLAB Code

https://drive.google.com/file/d/1YCLF879Io6ygF7tXPoS7pFwbMpb3-xpC/view?usp=sharing